## Assignment \#5 - Digital Logic Design I - Combinational Logic

CDA 3100, Computer Organization I

Problem 1 ( 40 points) Design a circuit that takes three bits X2, X1, X0, as input, and output one bit O as output. O is 1 if and only if $2<=\mathrm{X}<=5$ when $\mathrm{X}=(\mathrm{X} 2, \mathrm{X} 1, \mathrm{X} 0)$ is read as an unsigned integer.
(a) Construct the truth table.

| X2 | X1 | X0 | O |
| :--- | :--- | :--- | :--- |
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 1 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 0 |
| 1 | 0 | 0 |  |

(b) Write down the sum-of-product form of the function without any simplification in the form as: $\mathrm{O}=(\mathrm{X} 2 \& \mathrm{X} 1 \& \mathrm{X} 0)|(\sim \mathrm{X} 2 \& \mathrm{X} 1 \& \mathrm{X} 0)| \ldots$ (this is NOT the answer).
$\mathrm{O}=(\sim \mathrm{X} 2 \& \mathrm{X} 1 \& \sim \mathrm{X} 0)|(\sim \mathrm{X} 2 \& \mathrm{X} 1 \& \mathrm{X} 0)|(\mathrm{X} 2 \& \sim \mathrm{X} 1 \& \sim \mathrm{X} 0) \mid(\mathrm{X} 2 \& \sim \mathrm{X} 1 \& \mathrm{X} 0)$
(c) Simplify the circuit using K-map.

(d) Write down the Verilog module for this circuit called module HW5P1 (X2, X1, X0, O).
module HW5P1 (X2, X1, X0, O);
input $\mathrm{X} 2, \mathrm{X} 1, \mathrm{X} 0$;
output O;
assign $\mathrm{O}=(\sim \mathrm{X} 2 \& \mathrm{X} 1) \mid(\mathrm{X} 2 \& \sim \mathrm{X} 1)$;
endmodule
(e) ${ }^{* * * * A}$ Verilog code has been provided for this homework with an empty HW5P1 module. Please replace this module with your module and run simulation. Copy and paste the waveform only related to this problem here. (One way to copy and paste is (1) print screen (2) paste the screen to the windows paint program (3) crop the waveform (4) paste it here.).


Problem 2 (40 points) Design a circuit that takes three bits A, B, S as input, and output one bit $O$ as output. If $S=0, O=A \mid B$. If $S=1, O=A \& B$.
(a) Construct the truth table.

| S | A | B | O |
| :--- | :--- | :--- | :--- |
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 1 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 0 |


| 1 | 1 | 1 | 1 |
| :--- | :--- | :--- | :--- |

(b) Write down the sum-of-product form of the function without any simplification in the form as: $\mathrm{O}=(\mathrm{A} \& \mathrm{~B} \& \mathrm{~S})|(\mathrm{A} \& \mathrm{~B} \& \sim \mathrm{~S})| \ldots$ (this is NOT the answer)

$$
\mathrm{O}=(\sim \mathrm{S} \& \sim \mathrm{~A} \& \mathrm{~B})|(\sim \mathrm{S} \& \mathrm{~A} \& \sim \mathrm{~B})|(\sim \mathrm{S} \& \mathrm{~A} \& \mathrm{~B}) \mid(\mathrm{S} \& \mathrm{~A} \& \mathrm{~B})
$$

(c) Simplify the circuit using K-map.

(d) Write down the Verilog module for this circuit called module HW5P2 (S,A,B,O).
module HW5P2 (S,A,B,O);
input S, A, B;
output O;
assign $\mathrm{O}=(\sim \mathrm{S} \& \mathrm{~A})|(\mathrm{A} \& \mathrm{~B})|(\sim \mathrm{S} \& \mathrm{~B})$;
endmodule
(e) ${ }^{* * * * A}$ Verilog code has been provided for this homework with an empty HW5P2 module. Please replace this module with your module and run simulation. Copy and paste the waveform only related to this problem here. (One way to copy and paste is (1) print screen (2) paste the screen to the windows paint program (3) crop the waveform (4) paste it here.).


Problem 3 (10 points) Design a comparator which has 6 input bits, A2, A1, A0 and B2, B1, B0, and one output bit O . Let $\mathrm{A}=(\mathrm{A} 2, \mathrm{~A} 1, \mathrm{~A} 0)$ and $\mathrm{B}=(\mathrm{B} 2, \mathrm{~B} 1, \mathrm{~B} 0)$, and regard them as unsigned integers. If $A>B, O=1$; else $O=0$. Please write down the logic function as your answer. No Verilog code or simulation is needed. Hint: This problem should be solved by analyzing the underlying logic and no K-map is needed. For example, if $\mathrm{A} 2=1$ and $\mathrm{B} 2=0$, O must be 1 and there is no need to check the values of other input bits.
[Ans.] The key is the less significant bits matter only when the more significant bits are the same. So,
$\mathrm{O}=(\mathrm{A} 2 \& \sim \mathrm{~B} 2)\left|\left(\sim\left(\mathrm{A} 2^{\wedge} \mathrm{B} 2\right) \& \mathrm{~A} 1 \& \sim \mathrm{~B} 1\right)\right|((\sim(\mathrm{A} 2 \wedge \mathrm{~B} 2) \&(\sim(\mathrm{~A} 1 \wedge \mathrm{~B} 1) \& \mathrm{~A} 0 \& \sim \mathrm{~B} 0))$

Problem 4 ( 10 points) We talked about multiplexors in the class. A 4-1 multiplexor has 6 inputs: $\mathrm{S} 1, \mathrm{~S} 0, \mathrm{~d} 3, \mathrm{~d} 2, \mathrm{~d} 1$, and d0. It has one output, O . It works as follows. If $\mathrm{S} 1 \mathrm{~S} 0=00, \mathrm{O}=\mathrm{d} 0$. If $\mathrm{S} 1 \mathrm{~S} 0=01, \mathrm{O}=\mathrm{d} 1$. If $\mathrm{S} 1 \mathrm{~S} 0=10, \mathrm{O}=\mathrm{d} 2$. If $\mathrm{S} 1 \mathrm{~S} 0=11, \mathrm{O}=\mathrm{d} 3$. Show how to use only one $4-1$ multiplexor (no any other gates should be used, including the inverter, meaning that the inputs to the 4-1 multiplexor can only come from $\mathrm{A}, \mathrm{B}, \mathrm{C}, 0$, and 1 ) to implement function $\mathrm{F}(\mathrm{A}, \mathrm{B}, \mathrm{C})$ with truth table as follows. Please finish the following figure, showing each input of the 4-1 multiplexor is connected to what input value, as your answer. No Verilog code or simulation is needed. Hint: For this problem, no K-map is needed.

| A | B | C | Output |
| :--- | :--- | :--- | :--- |
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 0 |
| 0 | 0 | 1 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 0 |
| 1 |  | 1 | 0 |
|  |  |  |  |
|  |  |  |  |

[Ans.] Connect $\mathrm{S} 1<=\mathrm{A}, \mathrm{S} 0<=\mathrm{B}, \mathrm{d} 3<=0, \mathrm{~d} 2<=1, \mathrm{~d} 1<=\mathrm{C}, \mathrm{d} 0<=0$

| A | - |  |
| :--- | :--- | :--- |
| B | - |  |
| 0 | - |  |
| S 1 |  |  |
| S 0 |  |  |
| d 3 |  |  |
| C 3 | O |  |
| C 2 |  |  |
| C 1 |  |  |
| 0 | - |  |
| d 0 |  |  |

