Digital Circuits

Review – Getting the truth table

- The first step in designing a digital circuit usually is to get the truth table.
- That is, for **every** input combination, figure out what an output bit should be, and write them down in a table.

Review – From the truth table to circuits

- Any truth table can be translated into a circuits consisting of several **and** gates followed by one **or** gate.
  - It means that any function can be implemented in this way
- Call a row in the truth table in which the output is `1' a ``true row'' and the input combination in this row a ``true input combination'' or just a ``true combination.''
- Each **and** gate corresponds to one ``true row'' in the truth table. The **and** gate should output a `1' if and only if the input combination is the same as this row. If all other cases, the output of this **and** gate is `0.'
  - So, whenever the input combination is the same as one of the ``true combinations,'' one of the **and** gates outputs ``1'', so the output of the **or** gate is 1.
  - If the input combination is not the same as any of the ``true combinations,'' none of the **and** gates will output a ``1'', so the output of the **or** gate is 0.

Logic Functions

- Drawing circuits is … Usually we express logic functions using logic equations which are more succinct and carry the same information
  - The OR operator is written as +, as in A + B.
  - The AND operator is written as ·, as A · B.
  - The unary operator NOT is written as $\overline{A}$ or A'.
- **Remember:** This is **NOT** the binary field. Here 0+0=0, 0+1=1+0=1, **1+1=1.**

Logic functions

- For example, the sum in the one-bit full adder is

$$\text{Sum} = (a \cdot \bar{b} \cdot \overline{\text{CarryIn}}) + (\bar{a} \cdot b \cdot \overline{\text{CarryIn}}) + (\bar{a} \cdot \bar{b} \cdot \text{CarryIn}) + (a \cdot b \cdot \text{CarryIn})$$

- From a logic function you can immediately know what the circuit looks like.
- Truth table == Circuits == Logic function, equivalent.
- So we are going to get familiar with getting the logic functions from the truth table

Problems

- Ex 1. Assume that X consists of 3 bits, x2 x1 x0. Write a logic function that is true if and only if X contains only one 0

EX 1

X2	X1	X0	output
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

EX 1

X2	X1	X0	output
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

Ex 1

- Output = $x2x1x0' + x2x1'x0 + x2'x1x0$

Ex 2

- Assume that X consists of 3 bits, x2 x1 x0. Write a logic functions that is true if and only if X contains an even number of 0s.

EX 2

X2	X1	X0	output
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

EX 2

X2	X1	X0	output
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

Ex 2

- Output = $x2x1'x0' + x2'x1'x0 + x2'x1x0' + x2x1x0$

Ex 3

- Assume that X consists of 3 bits, x2 x1 x0. Write a logic functions that is true if and only if X when interpreted as an unsigned binary number is no less than 5.

Ex 3

X2	X1	X0	output
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

Ex 3

X2	X1	X0	output
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

Ex 3

- Output = $x2x1'x0 + x2x1x0' + x2x1x0$

The Karnaugh Map

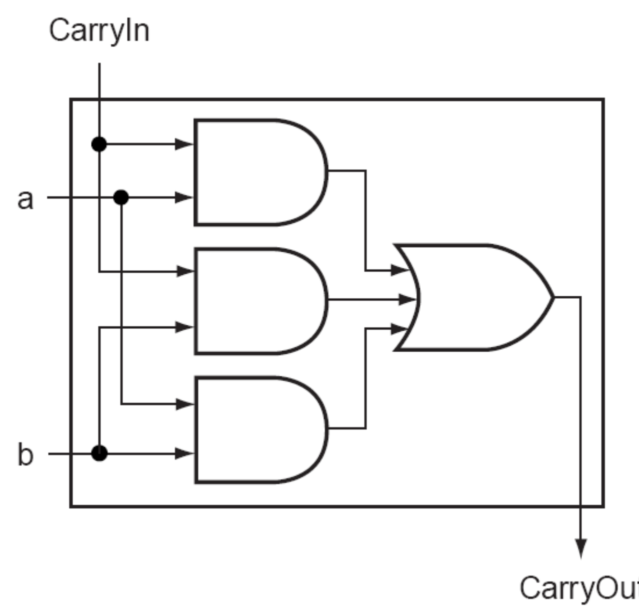
Simplifying Digital Circuits

- Reconsider the 1-bit full adder. The carry bit is

$$CO = (a \cdot b \cdot \bar{c}) + (a \cdot \bar{b} \cdot c) + (\bar{a} \cdot b \cdot c) + (a \cdot b \cdot c)$$

- But we can implement the function with a much simpler circuit:

$$CO = (a \cdot b) + (b \cdot c) + (c \cdot a)$$



- How to get there?

Simplifying digital circuit

- There are many methods.
  - Using boolean algebra
  - Using K-map
  - By just being really smart…

Boolean Algebra Laws

- Identity Law:
  - $A + 0 = A$
  - $A * 1 = A$
- Zero and One Laws:
  - $A + 1 = 1$
  - $A * 0 = 0$
- Inverse Laws:
  - $A + A = 1$
  - $A * A = 0$
- $A * A = A$

- Commutative Laws:
  - $A + B = B + A$
  - $A * B = B * A$
- Associative Laws:
  - $A + (B + C) = (A + B) + C$
  - $A * (B * C) = (A * B) * C$
- Distributive Laws:
  - $A * (B+C) = (A*B) + (A*C)$
  - $A + (B*C) = (A+B) * (A+C)$
- $A + A = A$

Boolean Algebra

- To use Boolean algebra, note that CO= abc’ + ab’c + a’bc + abc
- Now,
  - abc’+abc=ab(c’+c)=ab.
  - ab’c+abc=ac(b’+b)=ac
  - a’bc+abc=bc(a’+a)=bc
  - We used term abc three times because abc=abc+abc+abc!

K-map

- It is actually more convenient to use K-map to simplify digital circuits.
- K-map is a very mechanical procedure. Nothing fancy.
- It basically uses two rules: A+A=A, and AB+AB'=A.

K-map

- K-map

c \ ab	00	01	11	10
0	0	0	1	0
1	0	1	1	1

- CO = ab + ac + bc

K-map rules

- Draw the K-map. Remember to make sure that **the adjacent rows/columns differ by only one bit**.
- According to the truth table, write 1 in the boxes.
- Draw a circle around a rectangle with all 1s. **The rectangle must have size 1,2,4,8,16…**Then, reduce the terms by writing down the variables whose values do not change.
  - For example, if there is a rectangle with two 1s representing ab'c' and ab'c, you write a term as ab'.
- Note that
  - **A term may be covered in multiple circles!**
  - **The rectangle can wrap-around!**
- Simplify to the simplest circuits possible:
  - The circle should be as large as possible.
  - Try to get the minimum number of circles, i.e., minimum number of terms in the equation.

K-map

- F=a'bc'+a'bc+a'b'c+ab'c

c \ ab	00	01	11	10
0	**0**	**1**	**0**	**0**
1	1	1	0	1

K-map

- F=a’bc’+a’bc+a’b’c+ab’c

c \ ab	00	01	11	10
0	0	1	0	0
1	1	1	0	1

- F=a’b+b’c

K-map

- F=a’bc’+a’bc+abc’+abc+a’b’c

c \ ab	00	01	11	10
0	**0**	**1**	**1**	**0**
1	1	1	1	

K-map

- $F = a'bc' + a'bc + abc' + abc + a'b'c$

c \ ab	00	01	11	10
0	**0**	**1**	**1**	**0**
1	1	1	1	

- $F = b + a'c$

K-map

- F=a’bc’d+a’bcd+abc’d+abcd+a’b’c’d+abcd’

cd \ ab	00	01	11	10
00				
01	1	1	1	
11		1	1	
10			1	

K-map

- F=a’bc’d+a’bcd+abc’d+abcd+a’b’c’d+abcd’

cd \ ab	00	01	11	10
00				
01	1	1	1	
11		1	1	
10			1	

- F=bd+a’c’d+abc