Finite State Machines

Review

- How to implement a “counter”, which will count as 0,3,1,4,5,7,0,3,1,……

Q2	Q1	Q0	D2	D1	D0
0	0	0			
0	0	1			
0	1	0			
0	1	1			
1	0	0			
1	0	1			
1	1	0			
1	1	1			

Review

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Q2	Q1	Q0	D2	D1	D0
0	0	0	0	1	1
0	0	1	1	0	0
0	1	0			
0	1	1	0	0	1
1	0	0	1	0	1
1	0	1	1	1	1
1	1	0			
1	1	1	0	0	0

Review

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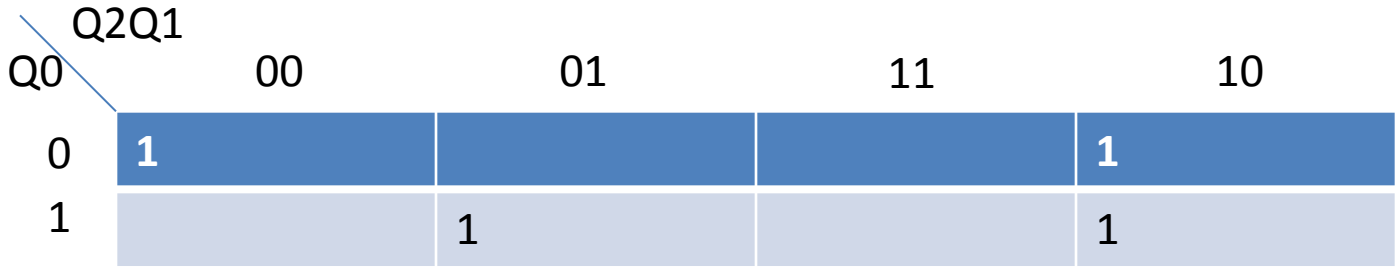
Q2	Q1	Q0	D2	D1	D0
0	0	0	0	1	1
0	0	1	1	0	0
0	1	0	X	X	X
0	1	1	0	0	1
1	0	0	1	0	1
1	0	1	1	1	1
1	1	0	X	X	X
1	1	1	0	0	0

Review

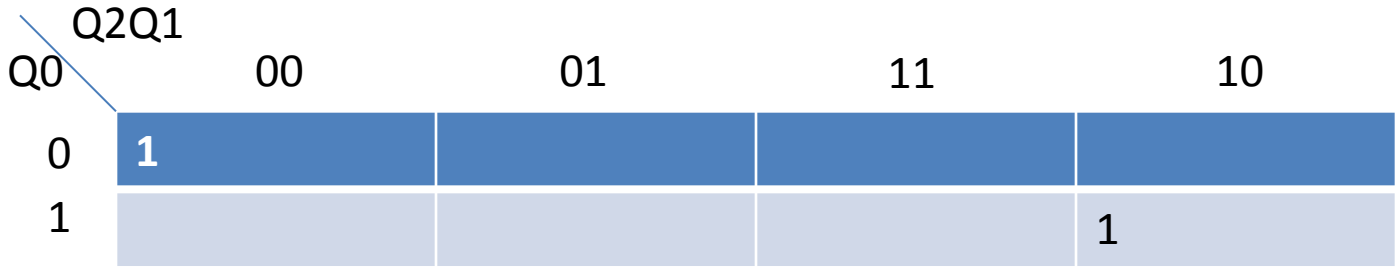
- D0 =



= (~Q1 & ~Q0) | (Q2 & ~Q1) | (~Q2 & Q1 & Q0)

Review

- D1 =

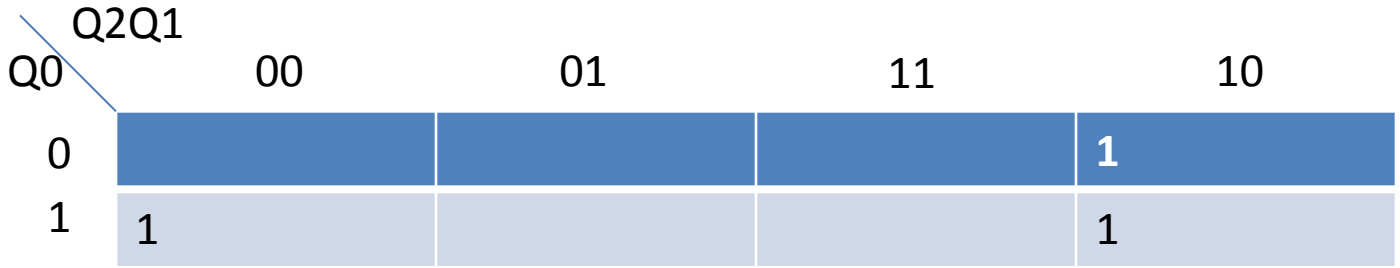


= (~Q2 & ~Q1 & ~Q0) | (Q2 & ~Q1 & Q0)

= ~Q1 & ~(Q2 ^ Q0)

Review

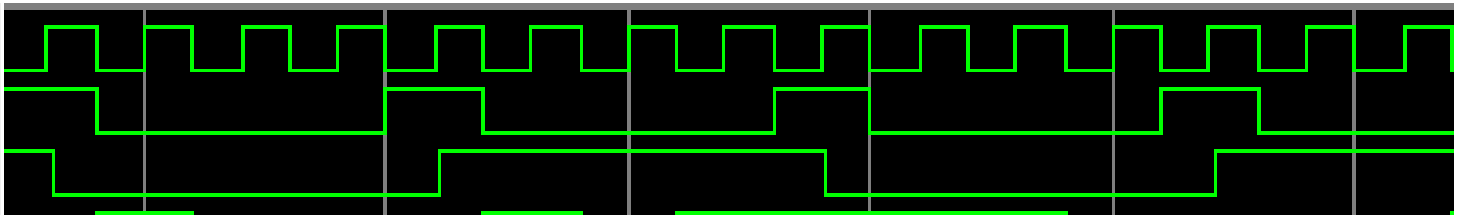
- D2 =



= (Q2 & ~Q1) | (~Q1 & Q0)

Parity checking

- Design a parity checking circuit that has one input X, and one output O.
- X may change every clock cycle. The change happens at the falling edge.
- The circuit samples the input at every rising edge of the clock. If the input is 1, consider as read a 1, else read a 0.
- O is 1 if all the bits read so far contains an odd number of 1s and 0 otherwise.



Parity checking

- Note that the output of the circuit depends on **ALL** past inputs.
- So one possible implementation is to remember all past inputs.
- Obviously bad…

Parity checking

- A better implementation is to “summarize” the past inputs into some “states.” For what we are concerned about,
  - Knowing the current state, the value of the output can be uniquely determined.
  - Given the current state, the future state transition does not depend on the past inputs.
- Note that
  - The states are just some binary numbers.
  - The number of states is significantly less than the number of input combinations, so we have a better circuit.

The difference from the counters

- Counters also have states. For example, the state of the 3-bit counters are 0,1,2,3,4,5,6,7.
- But counters have only the clk input, and is driven only by the clk. Knowing what the current state is, we know exactly what the next state should be.
- Here, obviously, the next state also depends on the input X.
- So we are moving to a more sophisticated example.

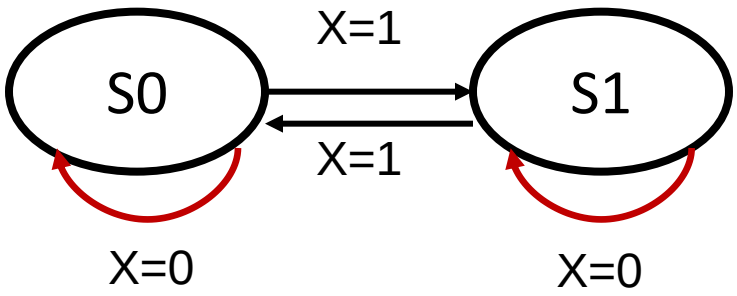
States

- Finding out what the states should be is a bit of art.
- Problems are different, so the solutions are also different.
- Experience will help.
- What should the states of the parity checking circuit be?

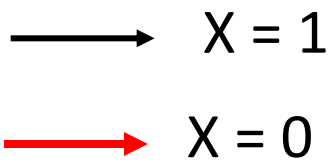
State

- The state is the parity of the bits read so far.
- Two states: S0 and S1.
  - S0: the bits have parity 0.
  - S1: the bits have parity 1.

State Diagram



- The state transition diagram.
  - Draw a circle around the state.
  - Draw arrows from one state to another.
  - Beside the arrows, show the values of the inputs that caused this transition.



Assign states

- Need to assign binary number representations to the states.
- Only one bit is needed. Let S0=0, S1=1.

Next State Function

Q	X	D
0	0	0
0	1	1
1	0	1
1	1	0

D = Q^X

Output function

- The circuit should generate the output.
- Clearly, the output function is O=Q.

Another FSM example – A sequence detector

- One input X, and one output O.
- X may change every clock cycle. The change happens at the falling edge.
- The circuit samples the input at every rising edge of the clock. If the input is 1, consider the read a 1, else read a 0.
- O is 1 (for one clock cycle, from positive edge to positive edge) if the last three bits read are 101.

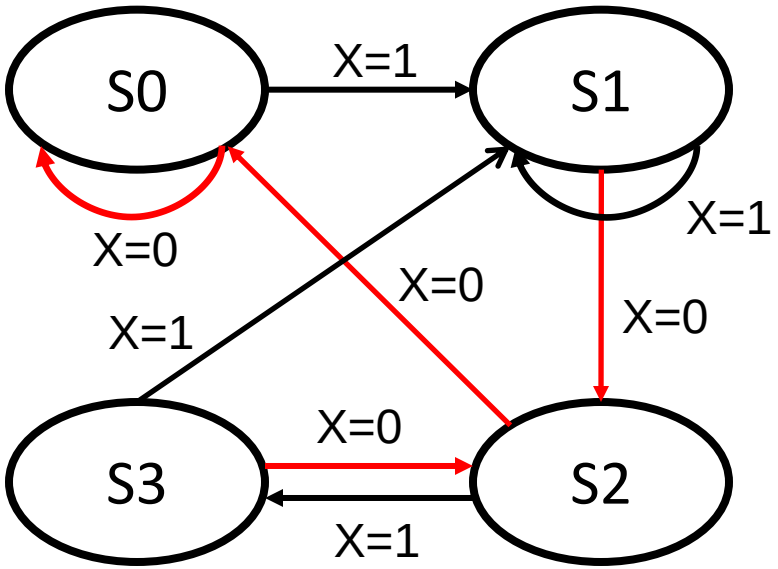
Suggestions?

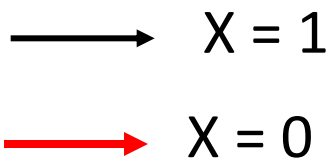
- Do we need to remember any states?
- What states do we need to remember?

Suggestions?

- Maybe we just connect 3 Dffs and output 1 if Q2Q1Q0=101?
- That is, we need to remember 8 states.
- Can do better than that.
- Remember what fractions of the sequence I have got.

4 states





- S0: got nothing. The initial state.
- S1: got 1.
- S2: got 10.
- S3: got 101.

Assign states

- S0 = 00
- S1 = 01
- S2 = 10
- S3 = 11

Next State Function

Q1	Q0	X	D1	D0
0	0	0		
0	0	1		
0	1	0		
0	1	1		
1	0	0		
1	0	1		
1	1	0		
1	1	1		

Next State Function

Q1	Q0	X	D1	D0
0	0	0	0	0
0	0	1	0	1
0	1	0	1	0
0	1	1	0	1
1	0	0	0	0
1	0	1	1	1
1	1	0	1	0
1	1	1	0	1

D1 = (Q0&~X)|(Q1&~Q0&X)

D0 = X

The output function

- Clearly, O = Q1&Q0.