Design the Following Circuit:

• A circuit has three inputs \((X_2,X_1,X_0)\)

• The output should be 1 if the three bits, regarded as an unsigned integer, is within \([2,5]\).
Step 1: Truth Table

• A truth table for n bits has $2^n$ rows. (All bit combinations are represented).

• The output is 1 if a bit pattern is included in the circuit. 0 otherwise.
## Truth Table

<table>
<thead>
<tr>
<th>$X_2$</th>
<th>$X_1$</th>
<th>$X_0$</th>
<th>Out</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
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<td>0</td>
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</table>
Step 2: Formula

• The standard way to represent the formula is the Disjunctive Normal Form (Sum of Products).

• Bits in every row with output 1 are used as inputs to an abstracted AND gate. (0 bits are negated).

• The outputs of the AND gates are inputs to an abstracted OR gate. Its output is the final output.
Step 2: Formula

Out = (¬X₂ & X₁ & ¬X₀) | (¬X₂ & X₁ & X₀) |  
     (X₂ & ¬X₁ & ¬X₀) | (X₂ & ¬X₁ & X₀)
Step 3: Circuit
AND gate using a selector (2-1 MUX)

• How do you implement an AND gate using a 2-1 multiplexer.

• 2-1 multiplexer:
  – 3 inputs: S, I₁, I₂.
  – If S=0, output=I₁
  – If S=1, output=I₂
AND gate using a selector (2-1 MUX)

• If you have inputs A and B to an AND gate,
  – Use A as $S$
  – Use 0 as $I_1$
  – Use B as $I_2$

• That is
  – $A = 0$, you go to $I_1$ which will output 0.
  – $A = 1$, you go to $I_2$ which will output B.
AND gate using a selector (2-1 MUX)