Term Project Report

CIS 5930/ISC 5932: Monte Carlo Methods

Spring 2013

A Review of Monte Carlo Methods in Real Estate

Author:
Sharanya Jayaraman

Instructor:
Dr. Michael Mascagni

May 2, 2013

Contents

1 Introduction **3**

1.1 Why Monte Carlo? 4

1.2 Real vs Financial Assets 4

2 The Notion of Equity **5**

2.1 Constant Elasticity of Variance 6

3 Discounted Cash Flow **6**

4 Adjusted Present Value **8**

5 Mortgage Pricing **8**

5.1 Mortgage Refinancing 10

6 Some examples **11**

Abstract

Monte Carlo Methods have been used in finance since the 1960's to simulate the various sources of said uncertainty that affect the value of instruments, portfolios or investments to calculate a value. This value is proportional to the likelihood of the events that affect it. Monte Carlo in finance is essentially a risk neutral evaluation. One of the main applications of Monte Carlo Methods in finance is Real Estate valuation. Estimating the value of real property is important to a variety of endeavors, including real estate financing, listing real estate for sale, investment analysis, property insurance and the taxation of real estate. The valuation models developed for financial assets are applicable for real assets as well. Since the market value of a property depends heavily on the values of surrounding properties and other factors in the neighborhood (like proximity to schools, safety of the neighborhood, etc.) it is possible to model them as a stochastic differential equation, and simulate using Monte Carlo methods.

Any attempt to predict the value of a property is curbed by the levels of uncertainty introduced by the different parameters in consideration. This makes Monte Carlo one of the better ways to estimate market values at some point in the future before deciding on an investment. This project introduces the notion of equity from the perspective of real estate and reviews the different methods used in real estate valuation, including the Discounted Cash Flow (DCF) model and the Adjusted Present Value (APV) model and the ways in which Monte Carlo methods are applied to these models. A Generalized Method of Moments estimator can also be put to use here. We also take a look at a Monte Carlo approach to Mortgage Pricing and the ways it is applied to Optimal Mortgage Refinancing and pricing when borrower default costs are unavailable or not observable.

1 Introduction

Monte Carlo methods are a broad class of computational algorithms that solve a problem by generating suitable random numbers and observing that fraction of the numbers obeying some property or properties. The method is useful for obtaining numerical solutions to problems which are too complicated or too unpredictable to solve analytically.[1]

In finance, Monte Carlo the use of methods were first suggested in 1964 by David B. Hertz through his Harvard Business Review article, Risk Analysis in Capital Investment; discussing their application in Corporate finance. In 1977, Phelim Boyle first used simulation in derivative valuation in his paper Options: A Monte Carlo Approach. The current scope of Monte Carlo Methods in finance has extended to include valuation and analysis of instruments, portfolios and investments. The areas in finance that use Monte Carlo methods include corporate finance, equity and option valuation, portfolio evaluation, risk and sensitivity analysis, project finance, capital investment etc. This covers option pricing, valuation of fixed income instruments and interest rate derivatives, personal financial planning, discrete even simulation, etc.

Real Estate was a comparatively late arrival to the Monte Carlo scene. However, the trend was pretty quick to catch on. Most of the analysis in real estate is based on the market value of the property in question. The market value is largely stochastic in nature. It depends on the market value of the properties nearby to an extent. So, properties in good neighborhoods have better values. This gives a Markov chain structure to the market value. However, this is not the deciding factor in the value of the property. The market value fluctuates based on a lot of factors, like the equity of the property, the current market situation, the intended and current use of the property, the valuation method, the financing used to buy the property, the initial investment, income produced by the property, etc. These are just the basic factors in consideration and adding more just results in increasing the complexity of the model.

[1] Weisstein, Eric W. "Monte Carlo Method." From MathWorld–A Wolfram Web Resource. http://mathworld.wolfram.com/MonteCarloMethod.html

1.1 Why Monte Carlo?

There are several analytic models which perform satisfactorily for real estate valuations. However, these models are fairly rigid. The operate under a strict set of constraints. Also, in order to reduce the complexity of these models, they are allowed to operate under a large amount of assumptions. They also fail to account for any surprises and uncertainties that arise in any situation that involves money and people. Thus, while adequate under perfect conditions, they fail to provide accurate predictions in the less-than-perfect real world scenarios. There will almost always be inconsistencies in the predicted and observed values when we use the analytical models.

Monte Carlo methods, on the other hand, thrive in situations where uncertainty is a somewhat dominant factor. Their very nature injects a level of randomness into the system, which can be exploited to account for some of the unpredictability of the consequences of human actions and other environmental factors. The results of Monte Carlo methods are not single value estimates but a stream of possible values moving from the worst to the best estimate, which allows a user to make and informed decision regarding the investment as the risk involved is factored into the model. The values predicted by the Monte Carlo models are always closer to the real world values.

1.2 Real vs Financial Assets

There are several common features in real and financial assets. Their values are determined by the cash flows they generate, the uncertainty associated with these cash flows and the expected growth in the cash flows. If all other factors are the same, it can be understood that the higher the level and growth in the cash flows and the lower the risk associated with the cash flows, the greater is the value of the asset. Having said that, there are also several differences in the way they operate. The difference in liquidity of the assets, the way they handle inflation and the type of investors involved are the primary discriminating factors. In particular, real estate investments often have finite lives and have to be valued accordingly. Many financial assets, such as stocks, have infinite lives. These differences in asset lives manifest themselves in the value assigned to these assets at the end of the estimation period. When we apply methods used for financial assets, like cash flow models to real assets, we must allow for some modification to accommodate these differences.

2 The Notion of Equity

Real assets and financial assets are also different in the way they define and handle equity. By definition, equity is the difference between the value of an asset and the liabilities held by the investor on the asset. The concept of equity used in real estate is the equity of redemption. The equity of redemption refers to the right of a mortgagor in law to redeem his or her property once the liability secured by the mortgage has been discharged. Today, most mortgages are granted by statutory charge rather than by a formal conveyance, although theoretically there is usually nothing to stop two parties from executing a mortgage in the more traditional manner. Equity of redemption is thus valued at the difference between the market price of the property and the amount of any mortgage or other encumbrance.[2]

There are two basic models that are used to estimate the cost of equity for financial assets: the capital asset model and the arbitrage pricing model. In both models, the risk of any asset, real or financial, is defined to be that portion of that assets variance that cannot be diversified away.[3] This non-diversifiable risk is measured by the market beta in the capital asset pricing model and by multiple factor betas in the arbitrage pricing model. The primary assumptions that both models make to arrive at these conclusions are that the marginal investor in the asset is well diversified and that the risk is measured in terms of the variability of returns. If we do so, however, we are assuming, as we did with publicly traded stocks, that the marginal investor in real assets is well diversified.

There are many factors that affect the market price that have to be taken into account. Some might exhibit Markov Chain characteristics. Some of the factors that affect the equity of a property include the market values, the clogs or impediments in the way of repaying the liability, the collateral hypothecated in the prime brokerage transaction, the equity of redemption itself, etc. The Equity of Redemption can be modeled as a Stochastic Differential Equation with the Constant Elasticity of Variance model.

[2] http://en.wikipedia.org/wiki/Equity_of_redemption

[3] http://people.stern.nyu.edu/adamodar/pdfiles/valn2ed/ch26.pdf

2.1 Constant Elasticity of Variance

The Constant Elasticity of Variance (CEV) model is a stochastic volatility model that attempts to capture the leverage effect.
The CEV model describes a process which evolves according to the following stochastic differential equation:

$$dS_t = \mu S_t dt + \sigma S_t^{\gamma} dW_t \tag{1}$$

- This is the Black-Scholes model where the price of the underlying asset follows a geometric Browninan Motion, denoted here by 'W'.
- The parameter γ controls the relationship between volatility and price, and is the central feature of the model.
- When γ <1 we see the leverage effect, commonly observed in equity markets; where the volatility of an asset increases as its price falls.

3 Discounted Cash Flow

The Discounted cash flow(DCF) analysis is a method of valuing an asset using the concepts of the time value of money. The DCF model attempts to attach a value to a property based in cash flows. All future cash flows are estimated and discounted to give their present values, and the sum of all future cash flows, both incoming and outgoing, is the net present value (NPV), which is taken as the value or price of the cash flows in question. Using the DCF model to compute the NPV takes as input cash flows and a discount rate and gives as output a price; the opposite processtaking cash flows and a price and inferring a discount rate, is called the yield. The factors involed in this model are:

- The cash inflow is given by rent. Rent in general refers to the monthly amounts paid by lease holders to the investor, but can be expanded to include any return on investment of capital on land during the time period under consideration. Rent is modeled by

$$Rent_t = \eta_t * Rent_t \tag{2}$$

- The cash outflow is given by expenses. This includes any amount spent or extra investment on the property during the time period under consideration including items such as property taxes, insurance, repairs and maintenance and advertising. Expenses is modeled by

$$Wk_t = \kappa_t * Wk_t \tag{3}$$

- Expected Growth: To estimate future cash flows, we need estimates of the expected growth rate in both rents/leases and expenses. A key factor in estimating the growth rate is the expected inflation rate.

- The Total Cash Flow for the system is given by

$$FCF_T = (1-\tau)(\eta_T * Rent_t - Exp_T - Wk_t) + \tau Dep_T + P_T - \tau * PV \tag{4}$$

Table 1: DCF applied to different types of properties

Property	Period	Annual return	Mean return	Standard Deviation	Serial correlation
CREF (commercial)	1969-87	10.80%	10.90%	2.60%	43.00%
REIT (commercial)	1972-99	14.20%	15.70%	15.40%	11.00%
I&S (commercial)	1960-69	8.70%	8.70%	4.90%	73.00%
C&S (residential)	1970-89	8.50%	8.50%	3.00%	17.00%
HOME (residential)	1947-89	9.80%	9.80%	4.70%	54.00%
HARRIS (residential)	1926-89	8.50%	8.50%	5.40%	55.00%
FARM (farmland)	1947-89	9.90%	9.90%	7.80%	64.00%

DCF model makes some deterministic assumptions. For example, the discount rate is assumed to be constant. We know from observation that the discount rate is not constant over any significant period of time. Making such assumptions leads to serious errors in the DCF model's predicted values.

Another drawback of the DCF method is that there is a circularity problem when part of the asset is financed by debt. This means that the values fed into the system are erroneous since the DCF model does not have any provisions for recovering from this case. It will assume that the property in question is healthy when it's actually not, resulting in predictions disparate from the real-world values.

4 Adjusted Present Value

The Adjusted Present Value (APV) model is proposed to overcome the drawbacks of the DCF model. The APV methodology proposes that a property has a value under perfect market conditions plus, possibly, an additional value resulting from market imperfections. Considering among market imperfections only the debt financing and using forecasted cash flows for a finite time horizon, the value of a property can be written as follows:

$$PV_0 = \sum_{t=1}^{T} \frac{FCF_t}{(1+k_u)^t} + \sum_{t=1}^{T} \frac{k_i * \tau * D_{t-1}}{(1+k_i)^t} + \frac{TV_T}{(1+k_u)^T}$$

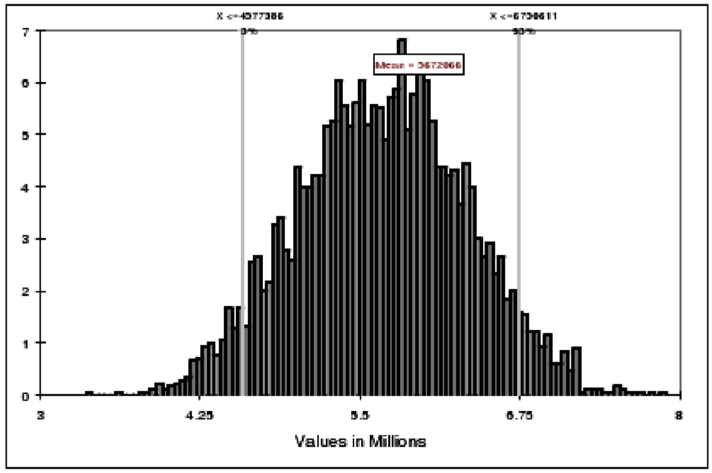
where

PV_0	= value of the property at time t=0
FCF_t	= free cash-to-property at time t (t = 1 to T)
D_t	= value of debt at time t
TV_T	= terminal value at time T
k_u	= cost of capital for a fully equity-financed property
k_i	= pre-tax cost of debt
τ	= tax rate

The advantage of equation above the standard DCF formula with the average cost of capital as the discount rate is that it considers the debt financing effects separately and consequently resolves the circularity problem. Moreover, the free cash flows are discounted at a rate that can be obtained from pension funds, as such investors in many countries (including Switzerland) buy properties without any leverage. Figure 1 shows the distribution of the NPV of a building using the APV model.

5 Mortgage Pricing

Monte Carlo methods can also be applied to Mortgage pricing. Mortgage lenders' offers of different interest rates and loan terms to different borrowers based on a grading of the credit worthiness of each borrower. Lenders grade borrowers, and offer different rates and terms to borrowers, based



Minimum	3335127
Mean	5672066
Maximum	7786397
Std Dev	644320
Skewness	-0.00715
Kurtosis	2.823

Figure 1: Distribution of the present value for a building

on several criteria including the borrower's credit score, payment history and the loan to value (LTV) ratio of the mortgage. Risk-based pricing is commonly used by Alt-A and subprime lenders. Risk-based mortgage pricing has expanded the types of mortgages lenders offer and increased the number of borrowers that can generally qualify for a mortgage. Alt-A and subprime mortgages, the types of mortgages generally subject to risk-based pricing, are frequently sold by the mortgage originator into the secondary mortgage market, where they typically become part of collateralized mortgage obligations (CMO), asset backed securities (ABS) and collateralized debt obligations (CDO). Risk-based pricing plays a large part in the structuring of CMO, ABS and CDO, enhancing their overallcredit ratingand making them attractive to wide range of investors. The process by which mortgage pricing is done is called the ruthless approach since it does not consider factors like the influence of default transaction costs on a borrower's debt.

The Monte Carlo approach to mortgage pricing is called the soft or fuzzy approach where the in-

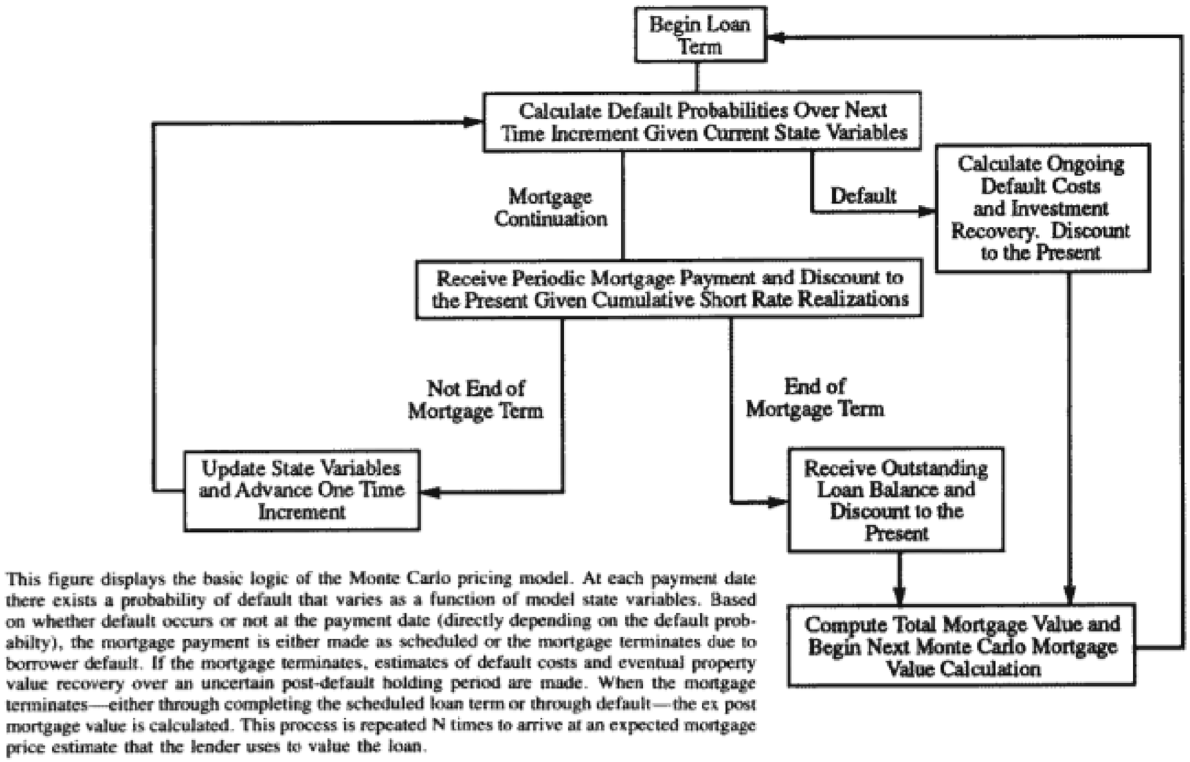


Figure 2: Monte Carlo Mortgage Pricing

vestor's uncertainty in the system is modeled as the probabilities of default, which can be estimated as a function of time and net equity. Then, if a default occurs, the severity of loss is calculated based on expected property value net of foreclosure costs and the time until the asset is actually sold. This allows for the simulation to closely follow the prices, default frequencies and the severity levels to emulate the ones actually found in the marketplace. Figure 2 shows the flowchart for Monte Carlo based mortgage pricing.

5.1 Mortgage Refinancing

Monte Carlo methods can also be used to make refinancing decisions. In real financial market, the consideration of discounted payment with matching the principal and interest rate method is more relevant and applicable to most of the industry practitioners. We can run simulations to see when and under what conditions would it be optimal to refinance a mortgage. From the simulations run by the authors of [8] it is clear that the debtors should refinance as earlier as possible when the lending rate is relatively low. Since mortgages can be considered to be options themselves, option pricing models can also find applications here.

6 Some examples

Continuum Analytics have a package called NumbaPro which is a CUDA API in Python, which is capable of simulating some Monte Carlo applications. [4] These are some example runs.

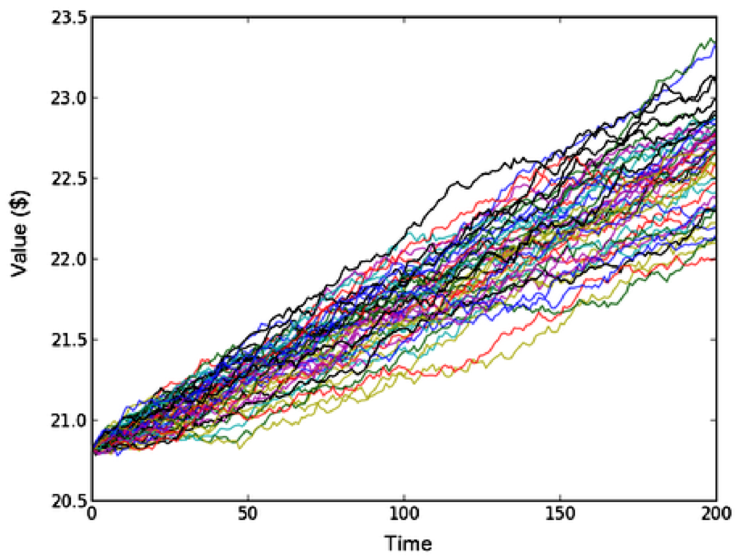


Figure 3: Simulated paths of a value of an asset using Monte Carlo

[4] http://docs.continuum.io/numbapro/index.html

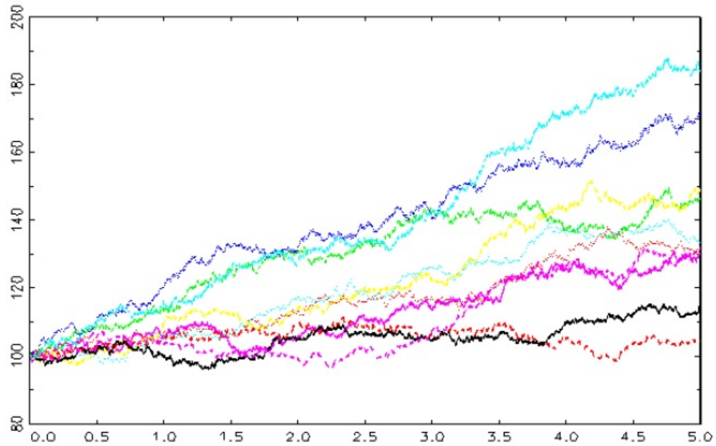


Figure 4: Yet another simulation

Conclusion

Monte Carlo Methods are the mainstay of the class of algorithms that deal with unpredictability. This is one reason they have dominated financial computations ever since the inception of simulations centered on finance. Though initially put to use in option pricing and risk analysis, Monte Carlo methods have gained a significant foothold in real estate valuation as well. The notion of equity in real estate gives rise to two methods of predicting the "safety" of an investment: we can either model the way cash flows through the system, or we can model the way the value of the property fluctuates. Either way, Monte Carlo methods have the ability to predict a range of situations, allowing the user to make an informed decision about the investment. They also play a significant role in pricing mortgages. Since they can model factors other than ruthless default criteria that affect borrower default decisions, a good method for recognizing the risks involved in the mortgage and the decision about the time and the rate for refinancing can be developed. It can be concluded that using Monte Carlo methods to simulate financial situations is a much better way to model real world situations in comparison with analytical models that do not factor unpredictability.

References

[1] Glasserman, Paul. Monte Carlo methods in financial engineering. *Vol. 53. Springer*, 2003.

[2] Hoesli, Martin; Jani, Elion and Bender, Andre Monte Carlo Simulations for Real Estate Valuation *FAME Research Paper Series, International Center for Financial Asset Management and Engineering*, rp148, 2005.

[3] Giacotto, Carmelo and Clapp, John. Appraisal-Based Real Estate Returns under Alternative Market Regimes *Journal of the American Real Estate and Urban Economics Association, V20,1*, 1–24, 1992.

[4] Baroni, Michel; Barthelemy, Fabrice and Mokrane, Mahdi Monte Carlo Simulations versus DCF in Real Estate Portfolio Valuation *ESSEC Woking Papers, ESSEC Research Center*, DR 06002, 2006.

[5] Jost, Allen; Nelson, Jennifer; Gopinathan, Krishna and Smith, Craig. Real estate appraisal using predictive modeling. *U.S. Patent No. 5,361,201.*, Nov. 1994.

[6] Rodda, David T.; Lam, Ken and Youn, Andrew. Stochastic Modeling of Federal Housing Administration Home Equity Conversion Mortgages with Low-Cost Refinancing *Real Estate Economics 32.4*, 589–617, 2004.

[7] Riddiough, Timothy J.; Thompson, Howard E. Commercial Mortgage Pricing with unobservable Borrower Default Costs. *Journal of the American Real Estate and Urban Economics Association, V21,3*, 265–291, 1993.

[8] Zheng, Jin; Gan, Siwei; Feng Xiaoxia and Xe, Dejun. Optimal Mortgage Refinancing Based on Monte Carlo Simulation *International Journal of Applied Mathematics, 42:2*, 2012.