

Unsupervised Color Image Segmentation Based on Gaussian Mixture Model

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Abstract

A novel color image segmentation method based on finite Gaussian mixture model is proposed in this paper. First, we use EM algorithm to estimate the distribution of input image data and the number of mixture components is automatically determined by MML criterion. Then the segmentation is carried out by clustering each pixel into appropriate component according to maximum likelihood (ML) criterion. The advantage of our method lies in its ability of less relying on initialization and segmenting images in a totally unsupervised manner. Experimental results show that our segmentation method can obtain better results than other methods.

1. Introduction

Color image segmentation is useful in many applications. With the segmentation results, it is possible to identify regions of interest and objects in the scenes, which is very beneficial to the subsequent image analysis or annotation. For example, many communication tasks require high compression ratio to save network resource. One possible way to realize the higher compression ratio is to discriminate objects in an image and compress only the necessary objects for users. This makes image segmentation extremely important for providing the needed information.

Image segmentation is defined as the process of dividing an image into different regions such that each region is homogenous. For intensity images (i.e. those represented by point-wise intensity levels) segmentation, there exists three popular approaches: histogram threshold techniques [1], edge-based methods [2] and region-based techniques [3]. Recently, with the progress on the theory of Gaussian mixture models, image segmentation based on Gaussian mixture model has also become popular [4] [5] [6].

In the work of [4], the author proposed a color image segmentation method based on maximum likelihood (ML) estimation. The observed color image is considered as a mixture of multi-variant densities and the mixture parameters are estimated using the EM algorithm. The segmentation is completed by clustering each pixel into a component

according to the Maximum likelihood (ML) estimation. Experiment results show this method is useful and stable in color image segmentation. However, a main drawback of this method is that the number of Gaussian mixture components is assumed known as prior, so it cannot be considered as totally unsupervised image segmentation method. Another problem in using EM algorithm in image segmentation is that the problem of mixture parameter initialization. This will greatly affect the segmentation result. A commonly used solution is initialization by randomly sampling in the mixture data [7] [8]. Although this method can result in fine segmentation result when the selected number is large, but the computation time is also heavily increased. When the selected number is small, it is high likely that some small regions may not be sampled, so the segmentation result is coarse.

In our proposed method, we use K-means algorithm to initialize the Gaussian mixture parameters and successfully solve the initialization problem. The number of mixture components is automatically determined by implementing the Minimum Message Length (MML) criteria into the EM algorithm. Therefore, the method can integrate the parameter estimation and model selection in a single algorithm, thus the method is totally unsupervised.

The rest of paper is organized as follows: in section 2, we review the finite Gaussian mixture model and propose our “EM+MML” algorithm; in section 3, the complete segmentation method is given; in section 4, experimental results will be presented to access performance of our method; the conclusion is given in the final section.

2 Gaussian Mixture Models and EM Algorithm

2.1 Gaussian Mixture Models

Consider a mixture model with $M > 1$ components in \mathfrak{R}^n for $n \geq 1$:

$$p(x|\theta) = \sum_{m=1}^M \alpha_m p(x|\theta_m), \forall x \in \mathfrak{R}^n \quad (1)$$

where $\alpha_1, \dots, \alpha_M$ are the mixing proportions, each θ_m is the set of parameters defining the m th component, and $\Theta \equiv \{\theta_1, \dots, \theta_M, \alpha_1, \dots, \alpha_M\}$ is the complete set of parameters needed to specify the mixture. Being probabilities, the α_m must satisfy

$$\alpha_m \geq 0, m = 1, \dots, M, \sum_{m=1}^M \alpha_m = 1. \quad (2)$$

For the Gaussian mixtures, each component density $p(x|\theta_m)$ is a normal probability distribution:

$$p(x|\theta_m) = \frac{1}{(2\pi)^{\frac{n}{2}} \det(\Sigma_m)^{\frac{1}{2}}} \exp\left\{-\frac{1}{2}(x - \mu_m)^T \Sigma_m^{-1} (x - \mu_m)\right\} \quad (3)$$

Here we encapsulate these parameters into a parameter vector, writing the parameters of each component as $\theta_m = (\mu_m, \Sigma_m)$, to get $\Theta = (\alpha_1, \alpha_2, \dots, \alpha_M, \theta_1, \theta_2, \dots, \theta_M)$. Then, (1) can be rewritten as

$$p(x|\theta) = \sum_{m=1}^M \alpha_m N(x|\mu_m, \Sigma_m) \quad (4)$$

where $N(x|\mu_m, \Sigma_m)$ is a Gaussian distribution with mean μ_m and covariance Σ_m .

2.2 EM algorithm

The commonly used approach for determining the parameters Θ of a Gaussian mixture model from a given dataset is to use the maximum-likelihood estimation. The EM algorithm is a general iterative technique for computing maximum-likelihood when the observed data can be regarded as incomplete. The usual EM algorithm consists of an E-step and an M-step. Suppose that $\Theta^{(t)}$ denotes the estimation of Θ obtained after the t th iteration of the algorithm. Then at the $(t+1)$ th iteration, the E-step computes the expected complete data log-likelihood function

$$Q(\Theta, \Theta^{(t)}) = \sum_{k=1}^K \sum_{m=1}^M \{\log \alpha_m p(x_k|\theta_m)\} P(m|x_k; \Theta^{(t)}) \quad (5)$$

where $P(m|x_k; \Theta^{(t)})$ is a posterior probability and is computed as

$$P(m|x_k; \Theta^{(t)}) = \frac{\alpha_m^{(t)} p(x_k|\theta_m^{(t)})}{\sum_{l=1}^M \alpha_l^{(t)} p(x_k|\theta_l^{(t)})} \quad (6)$$

The M-step finds the $(t+1)$ th estimation $\Theta^{(t+1)}$ of Θ by maximizing $Q(\Theta, \Theta^{(t)})$

$$\alpha_m^{(t+1)} = \frac{1}{K} \sum_{k=1}^K P(m|x_k; \Theta^{(t)}) \quad (7)$$

$$\mu_m^{(t+1)} = \frac{\sum_{k=1}^K x_k P(m|x_k; \Theta^{(t)})}{\sum_{k=1}^K P(m|x_k; \Theta^{(t)})} \quad (8)$$

$$\Sigma_m^{2(t+1)} = \frac{\sum_{k=1}^K P(m|x_k; \Theta^{(t)}) (x_k - \mu_m^{(t+1)}) (x_k - \mu_m^{(t+1)})^T}{\sum_{k=1}^K P(m|x_k; \Theta^{(t)})} \quad (9)$$

EM algorithm is highly dependent on initialization. A common approach is using multiple random starts and choosing the final estimate with the highest likelihood. This will greatly increase the computation burden. In our method, we initialize the mixture parameters by K-means algorithm.

2.3 MML criterion

Another main issue of using Gaussian mixture model in image segmentation is the difficulty to estimate the number of components. Many methods, such as the method used in [4], assume the number is known in advance. In other words, the number of segmentation region is determined in advance by user. Obviously, this is supervised or semi-supervised method.

In order to circumvent this problem, a model selection criterion, Minimum Message Length(MML) [9], is introduced in our algorithm. For an M-components mixture, the parameters specifying each components are expressed as:

$$\hat{\theta} = \arg \min L(\theta, y), \quad (10)$$

with

$$L(\theta, y) = \frac{N}{2} \sum_{m=1}^M \log\left(\frac{n\alpha_m}{12}\right) + \frac{M}{2} \log \frac{n}{12} + \frac{M(N+1)}{2} - \log p(y|\theta) \quad (11)$$

where N is the number of parameters specifying each component, n is the number of observed data, M is the number of components, and α, θ are the estimated gaussian mixture parameters. The EM+MML algorithm is presented in Figure 1.

The procedure in Figure 1 shows the whole ‘‘EM+MML’’ algorithm. Given a 3 channel RGB color image, the input data is the 3 dimensional RGB entry of each pixel. With the input data, the EM algorithm was run iteratively from M_{max} to M_{min} , in each iteration, the mixture parameters are initialized by K-means algorithm. The number of components and mixture parameters are selected according to the minimum message length expressed in eqn.(11).

- **Input:** Data vector X ;
- **Params:** Maximum and minimum number of components M_{max}, M_{min}
- **Procedures:**

Set the initial Message Length $L_{min} = +\infty$;

for $m = M_{max}:-1:M_{min}$

Initialize $\Theta_0 = \Theta_{k_means}$;

$\hat{\Theta}_m = \hat{\Theta}_{EM}$;

Compute L_m ;

if $L_m \leq L_{min}$, **then**

$L_{min} = L_m$; $\hat{\Theta}_{best} = \hat{\Theta}_m$;

end if

end for

- **Output:** Mixture model parameters $\hat{\Theta}_{best}$ and the number of mixture components M_{best} determined by MML.

Figure 1: EM+MML algorithm

3 The Segmentation Algorithm

The segmentation consists of two steps. The first step is mixture estimation. The above “EM+MML” method is used to estimate the mixture parameters and determine the number of Gaussian components.

The second step is pixel clustering. The segmentation is carried out by assigning each pixel into a proper cluster according to the Maximum likelihood(ML) estimation. The j -th element is labelled L_j according to the following equation,

$$L_j = \max_i \frac{\exp\{-\frac{1}{2}(x_j - \mu_i^{EM})(\Sigma_i^{EM})^{-1}(x_j - \mu_i^{EM})^T\}}{|\Sigma_i^{EM}|^{-\frac{1}{2}}} \quad (12)$$

where x is the input data, μ^{EM} , Σ^{EM} are the estimated mixture parameters respectively.

4 Experimental Results

In order to evaluate our proposed method, we also carried out image segmentation using the supervised EM method proposed in [4]. In the supervised method, the parameters is initialized by randomly sampling and the number of components are assumed known in advance. Fig.4 and Fig.7 shows the segmentation results of the well-known “house” and “woman” images by our “EM+MML” method.



Figure 2: Original “house” image

The estimated number of region is 15 and 8 respectively. Compared with the segmentation results using supervised algorithm(Fig.3 and Fig.6), we can see the images are segmented more correctly by using our proposed method.

5 Conclusion

This paper proposed an unsupervised color image segmentation method based on finite Gaussian mixture model. The observed color image was considered as a mixture of multi-variate normal densities. Each pixel entry is considered as a 3-dimensional input data of the mixture. The segmentation is carried out by the ML estimation.

The advantage of our method lies in two aspects. First, by using the K-means algorithm, we successfully circumvent the initialization problem of EM algorithm. Second advantage of the algorithm is that the number of segmentation region is automatically determined by the MML criterion, so the algorithm is called unsupervised.

Experimental results show the proposed method has better segmentation result than the compared supervised methods.

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References

- [1] C. A. Glasbey, “An Analysis of Histogram-based Thresholding Algorithm,” CVGIP:Graphical Model and Image Processing, Vol. 55, pp. 532-537, November 1993.



Figure 3: Segmentation result use supervised EM method, the pre-defined region number is 15



Figure 4: Segmentation result use our “EM+MML” method, the determined region number is 15



Figure 5: Original “woman” image



Figure 6: Segmentation result use supervised EM algorithm, the pre-defined region number is 6



Figure 7: Segmentation result using our “EM+MML” method, the determined region is 6

- [2] M. Brejl and M. Sonka, “Edge-based Image Segmentation: Machine Learning from Examples,” IEEE World Congress on Computational Intelligence, pp. 814-819, May 1998.
- [3] Leung. T and Malik. J, “Contour Continuity in Region Based Image Segmentation,” Proceedings of the Fifth Europe Conference on Computer Vision, June 1998.
- [4] T. Yamazaki, “Introduction of EM Algorithm into Color Image Segmentation,” Proceedings of ICIRS’98, pp. 368-371, August 1998.
- [5] H. Caillol and W. pieczynski and A. Hillion, “Estimation of Fuzzy Gaussian Mixture and Unsupervised Statistical Image Processing,” IEEE Transaction on Image Processing, Vol. 6, pp. 425-440, March 1997.
- [6] J. J. veeberk and N. Vlassis and B. Klose, “Greedy gaussian mixture model learning for tetxture image segmentation,” Proceedings of the Workshop on kernel and Subspace Methods for Computer Vision, pp. 37-46, August 2001.
- [7] G. McLanchlan and T.krishnan, The EM algorithm and extensions. New York: John Wiley and Sons, 1997.
- [8] G. Mclanchlan and D. Peel, Finite Mixture Models. New York: John Wiley and Sons, 2000.
- [9] M. A. Figueiredo and A. K. Jain, “Unsupervised Learning of Finite Mixture Models,” IEEE Transaction on Pattern Analysis and Machine Intelligence, Vol. 24, pp. 381-396, March 2002.